

Diameter of the Moon and the Ecliptic Limit

Early astronomers were able to determine the size of the moon using simple geometry. They knew the size of Earth from Eratosthenes' measurement (Chapter 2). These astronomers also knew that both Earth and the moon cast a cone-shaped shadow out into space. As the moon passed through Earth's shadow during a lunar eclipse, they noted that Earth's shadow was about 2.4 times the diameter of the Moon. In Part A, you will use the same methods the early astronomers used to determine the size of the moon.

The early astronomers also found that the moon's path through the sky can be as much as 5 degrees above or below the sun's path (the ecliptic). This meant that the moon's shadow would often end in space above or below Earth, rather than falling on Earth to cause a solar eclipse. For the same reason, the moon can pass above or below Earth's shadow, rather than passing through the shadow to cause a lunar eclipse. Eclipses can occur only when the moon is at or near the ecliptic. The part of the moon's orbit when this occurs is called the **ecliptic limit**. Although the moon crosses the ecliptic twice each month, eclipses do not occur that often. For an eclipse to occur, the moon must be in a new moon or full moon phase when it is in the ecliptic limit. In Part B, you will determine the average length, in kilometers, of an ecliptic limit.

Lab Skills and Objectives

- To **measure** the moon's diameter using the method of early astronomers
- To **identify** the portion of a lunar orbit where an eclipse could occur

Materials

- 2 sheets plain paper
- drawing compass
- pencil
- protractor
- metric ruler

Procedure

Part A

1. To determine the size of the moon, you will draw a model of Earth and the moon. In this model, imagine yourself as an observer out in space looking at Earth and the moon. To draw the model, turn a sheet of paper so the long side is toward you. Draw a thin line that divides the paper in half lengthwise. This thin line serves as a guideline for procedure steps 1, 2, and 3. Use the compass to draw a circle 4 cm in diameter centered on the guideline and close to one edge of the paper. (Remember that a diameter of 4 cm means a radius of 2 cm.) The circle represents Earth.

2. Using the edge of the ruler, draw a line from the top of the circle to the point where the guideline meets the far edge of the paper. Then draw a similar line from the bottom of the circle to the same point. Your drawing should now look something like an ice-cream cone with a scoop of ice cream in the end (Figure 24.1). The cone shape represents Earth's shadow out into space.

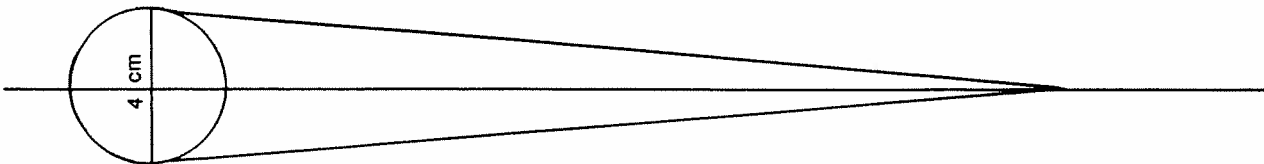


Figure 24.1

3. With the protractor, measure the entire angle at the point of the cone-shaped shadow. Measure carefully and estimate the size of the angle to the nearest tenth of a degree. Call your measurement angle A and record its value in the table in Figure 24.5.
4. Angle A is the angular width of Earth's shadow on your model. At the moon's distance from Earth, Earth's shadow is about 2.4 times as wide as the moon. Multiply angle A by 2.4 and record this value as angle B in Figure 24.5.
5. Angle B will be used to determine the width in kilometers of Earth's shadow. Divide angle B by two and record the value in Figure 24.5. Place your protractor at the point where your guideline crosses the circle of Earth. Measure half of angle B above the guideline and make a small mark. Then measure half of angle B below the guideline and make another small mark. Draw lines from the point where the guideline crosses Earth out to each of the small marks. This should form a second cone (Figure 24.2).

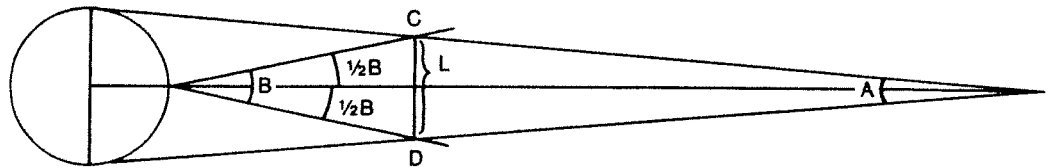


Figure 24.2

6. Label the points where the two lines of the second cone cross the lines of the first cone point C and point D. Draw a straight vertical line between the two points. Label the line L. L represents the average width of Earth's shadow at the distance of the moon's orbit. Measure the length of line L in centimeters and record the value in Figure 24.5.
7. Line L is 2.4 times the diameter of the moon. Divide L by 2.4. Call this value L' and record its length in Figure 24.5. L' is the diameter of the moon on the scale of your model.
8. In order to determine the fractional size of the moon by comparing it to the size of Earth, the size of Earth needs to be considered as the value 1. Since your original circle, Earth, was 4.0 cm in diameter, the cone-shaped shadow you drew is 4 times too large for this purpose. Divide L' by 4.0 and record the answer. L' divided by 4 is the fractional size of the moon's diameter compared to Earth's diameter. Use the value L' divided by 4 to answer Analysis and Conclusions questions 1 and 2.

Part B

9. Now you will construct another model to determine the length of an ecliptic limit. In this model, imagine yourself as the sun facing earth and the moon. (To simplify the drawing, everything in the model will be shown in the plane of the paper.) Take a new sheet of paper and hold it with the long side toward you. Mark a point 2.5 cm from the left (short) edge, and 10 cm from the top (long) edge. Label this point X. Use point X as the center of another 4-cm circle to represent Earth.
10. At the right side of the paper, measure and mark three points from the top edge of the paper. The three points should be at distances of 8, 10, and 12 cm from the top edge. Draw a line with the straightedge from the top edge of the circle to the top mark. Then draw a line from the center of the circle to the second mark. Finally, draw a third line from the bottom of the circle to the bottom mark. These three lines should be parallel (Figure 24.3). The center line represents Earth's orbit. The top and bottom lines represent the upper and lower edges of Earth's shadow in space. Although the shadow is actually cone-shaped, the cone is so long that for the small part shown in this model, the taper of the cone would not show.

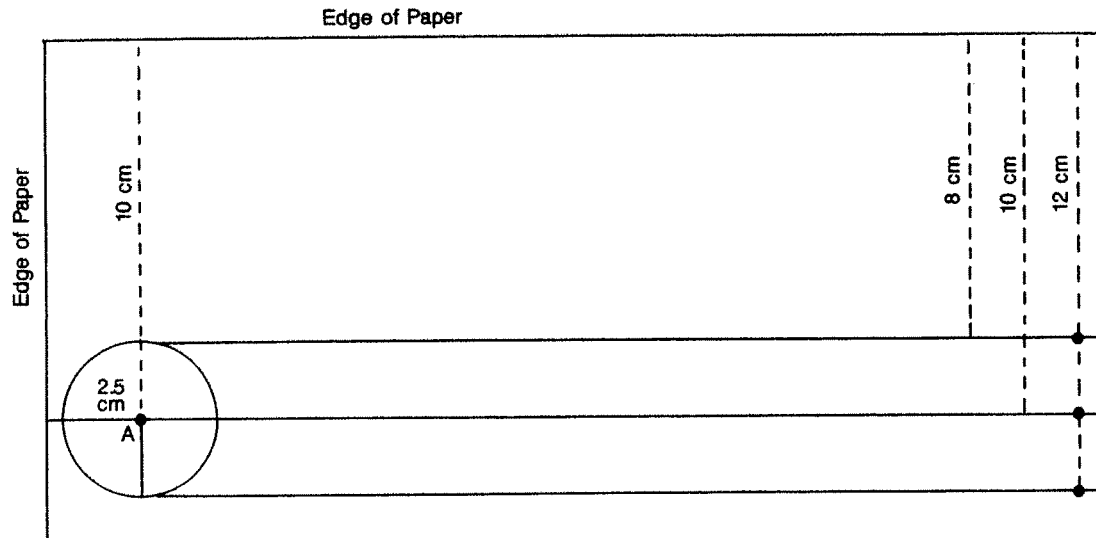


Figure 24.3

11. The moon can be above or below Earth's path around the sun by a maximum of 5 degrees. From the center of the circle, measure an angle of 5 degrees above the center line and make a small mark. Measure very carefully as a small error now will cause a large error later.

12. Carefully draw a line through the center of the circle and the mark to the edge of the paper. This line should cross the top parallel line just before the end of the paper (Figure 24.4). This line represents part of the moon's orbit that is tilted 5 degrees above the plane of Earth's orbit.

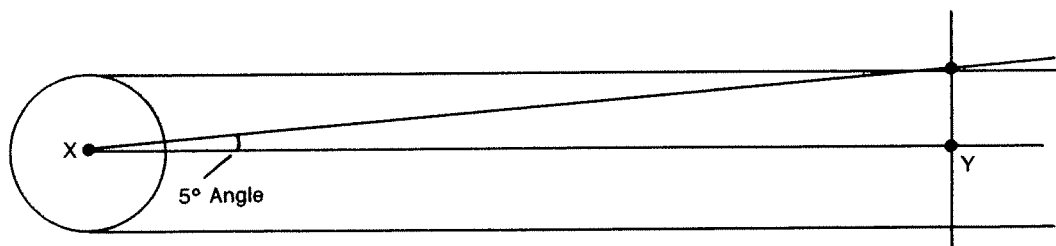


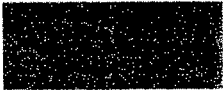
Figure 24.4

13. Draw a perpendicular line from the point where the 5-degree line crosses the top line down through the three parallel lines. Label the point where this line crosses the center line point Y.

14. Measure the distance from point X along the center line to point Y. Record this value in Figure 24.5. The line represents one-half the distance of the ecliptic limit. The other half of the ecliptic limit would extend off the paper to the left, to the point where the moon's orbit, tilted down 5 degrees, crosses the lower limit of Earth's shadow. Determine the total length of the ecliptic limit on your model by multiplying the length of line XY by 2. Record this value as XY' in Figure 24.5. Answer Analysis and Conclusions question 3.

Part A diameter of original circle (cm)	
angle A, to the nearest tenth of a degree	
angle B (angle A times 2.4)	
angle B divided by 2	
length of line L (cm)	
length of L' (L divided by 2.4)	
L' divided by 4	
Part B length of line XY (cm)	
XY' (length of line XY times 2)	

Figure 24.5



1. Using an Earth diameter of 12 800 km and your fractional diameter of the moon, determine the value for the moon's diameter based on your model. Set up the problem and show your work.

2. The moon's actual diameter is 3476 km. Determine the percent difference between your value for the moon's diameter and the moon's actual diameter. Show all work.

$$\text{Percent difference} = \frac{\text{difference between calculated and actual value}}{\text{actual value}} \times 100$$

3. Since Earth's diameter is 12 800 km, your model has a scale of 1 cm = 3200 km (12 800 km/4 cm). Determine the number of kilometers represented by the line XY'.

Your answer is the length in kilometers of the ecliptic limit.